

The Magic power of V squared

Several formulas used in aviation use the term V squared, or velocity squared. The significance of this becomes apparent as we discover that velocity affects performance in a big way.

The first two formulas are closely related. These are the formulas for lift and drag

$$\text{Lift} = \frac{1}{2} P V^2 S C_L$$

$$\text{Drag} = \frac{1}{2} P V^2 S C_D$$

Now before you get scared off, I'll explain – it's really pretty simple.

In each of these formulas we have P or rho which is simply the air density. Or its weight per cubic foot – how thick the air is.

S which is the square area of the wing or other surface – how much of that air we can affect

And the Co-efficient of lift or drag depending on what we're measuring. This value will change with things like the angle of attack and the shape of the airfoil. How much we're affecting the air.

Each of these factors will affect the lift or drag in direct proportion to their value. Half the air density – you get half the lift. Double the square area of the wing, you get twice the lift. And so on.

And then we have V squared. Which means, of course, that the velocity – or speed is multiplied by itself. If we double this value we get four times the lift (we also get four times the drag). Triple the value and we get 9 times. This is why for an airplane to go twice as fast, you basically need four times as much thrust. When we want to go faster we pay a steep penalty. Most airplanes can barely double their speed – their indicated airspeed - within their operating range.

On takeoff, as we just get off the ground and fly low waiting for the towplane to accelerate, as our speed increases, if we were to simply hold a constant pitch attitude and therefore a constant angle of attack, our lift will increase rapidly as the airspeed builds. For that reason as we accelerate, we'll need to lower the nose to maintain the same *altitude* while we wait for the towplane to be airborne.

As we go up in altitude, the air density decreases. To make up for this we fly a faster true airspeed which results in the same indicated airspeed for the same amount of lift. The nice thing is that since both the lift and drag are both affected proportionally to the air density – so our glide ratio remains the same regardless of altitude. However, we will have a higher true airspeed to achieve that same glide ratio

But at around 20,000 feet where the density is half of that at sea level, we **don't** have to go twice as fast to generate the same amount of lift since the impact of a small speed increase is so much greater. There, our true airspeed will only need to be about 35% faster.

By the way, did you know you can estimate your true airspeed by adding 2% of your indicated per thousand feet in altitude? At 10,000 feet, you're going roughly 20% faster than indicated. (a little less below 12,000 and a little more above it)

In another example, as we make a steep turn and tighten the turn up to a 45 ° angle, the outer wingtip is going faster than the inner wingtip – maybe 10 knots faster, but that small speed difference adds up to

more lift and drag on the outer wing forcing us to cross control in those tight turns to avoid an ever increasing bank angle.

Since this also works for drag as well as lift, if you want to get rid of energy, you'll get that same effect by going faster. We can increase our drag exponentially more by going faster. That exponentially higher drag will convert your excess altitude and airspeed into heat, noise, vibrations, and turbulence. Negative flaps help to reduce this effect by reducing the coefficient of lift but V^2 still wins out.

The next bit of magic concerns our kinetic energy. This is simply the energy of our motion.

The formula looks like this

$$KE = \frac{1}{2} M V^2$$

Again we have a direct relationship with one parameter, the mass, then there's the V squared again.

Therefore if we double our mass we have twice the kinetic energy. It takes longer to get going and takes longer and more effort to stop.

But, it's pretty hard to double your mass. There's certainly no way to do it in flight and even dropping a heavy water ballast load won't cut the glider's weight by half.

But a change in our velocity and that can make a big difference in how much energy we have.

This is how little rocks make big craters.

This is why it's hard and expensive to get a rocket into orbit. – because they need to change their velocity by a lot and that needs an exponential amount of energy to do it.

But let's explore a very important scenario where V squared plays an important role for gliders– landing distance. Specifically in a downwind landing scenario, because here we have the opportunity to make a big difference in the landing velocity at the same indicated airspeed.

Let's take a typical 1000 pound Schweizer 233.

We'll say that on a calm day we'll touch down around 40 miles per hour

Now we'll plug in some numbers, convert the English units to meters/second and kilograms to generate the kinetic energy value in joules

Zero wind 72,000 or 72 kilo joules

With a 10 mph headwind – 25 % slower and therefore a touchdown speed of 30 mph

Our kinetic energy is 40.7 kg 44% less energy

How about with a 20 mph headwind?

Now we're touching down at a groundspeed of 20 mph and a KE of only 18KJ half the speed and a quarter of the energy.

But if we were to do a return to the field and land downwind with that 20 mph wind, now we'll have a groundspeed of 40 +20 or 60 mph.

50% faster than the calm wind scenario for 2 ¼ times the energy

And 3 times faster than landing into that 20 mph wind for 9 times the energy. Wow!

Those higher energies are what you need to get rid of to stop the glider. This is why it quickly becomes a risky move to land downwind with a high tailwind.

Perhaps you've recognized what these formulas all have in common. They all are extrapolations of Isaac Newton's second law of motion $F=MA$

Or a force, (for example lift or drag, or the force required to stop a glider) is equal to the mass (whether it's the mass glider's mass or of the air flowing over the wing) times the acceleration. And what is an acceleration but a change in velocity over time or feet per second, per second = written as Velocity squared or V^2